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Veronika Furst* (furst@euclid.colorado.edu), Department of Mathematics, University of Colorado at Boulder, Campus Box 395, Boulder, CO 80309-0395. *Semi-orthogonal Parseval wavelets in abstract Hilbert spaces*. Preliminary report.

The complete characterization of orthonormal wavelets is known in $L^2(\mathbb{R})$. G. Gripenberg and X. Wang proved independently that ψ is a Parseval wavelet if and only if the two equations $\sum_{j \in \mathbb{Z}} |\widehat{\psi}(2^j \omega)|^2 = 1$ and $\sum_{j=0}^{\infty} \widehat{\psi}(2^j \omega) \overline{\widehat{\psi}(2^j(\omega + m))} = 0$, where $m \in 2\mathbb{Z} + 1$, are satisfied for a.e. $\omega \in \mathbb{R}$. In particular, ψ is an orthonormal wavelet if and only if $\|\psi\|_2 = 1$. Their results have been generalized to multiwavelets, to higher dimensional square-integrable function spaces, to dilation by a real expansive matrix with translation by a general lattice, and to dual frames.

Our objective is to find an analogous characterization of semi-orthogonal Parseval wavelets in an abstract Hilbert space H . We examine the structural assumptions that must be made about H in order to compensate for the loss of known facts about $L^2(\mathbb{R})$. While one direction of the proof is a natural analog of the classical case, the other direction provides complication. (Received September 06, 2005)