

1015-35-79

John B Butler* (hmkt@pdx.edu), P.O. Box 751, Dept. Math & Statistics, Portland, OR 97207.
On an Inverse Theory for Schrödinger's Operator in Momentum Space for Certain $L_2(R^3)$ Hölder Continuous Potentials Preliminary report.

Let $H = H^0 + V$ be the Schrödinger operator on $L_2(R^3)$ in the momentum representation $H^0 u = \rho^2 u, V u = \int v(k - k') u(k') dk', k = \rho \hat{k}, |\hat{k}| = 1, -\infty < \rho < \infty, \langle \hat{k} | \alpha \rangle \geq 0, \alpha = (0, 0, 1)$. K will be an integral operator $K u = \int K(k, k') u(k') dk', dk = d\rho d\hat{k} = d\rho \sin \theta d\theta d\phi, u(k) \in L_2(R^3 : d\rho d\hat{k}) = \mathbf{H}$. $\rho K(k, k') \rho'$ on \mathbf{H} corresponds to $K(k, k')$ on $L_2(R^3 : \rho^2 d\rho d\hat{k})$. We suppose a weight operator $I + \Omega$ given where $\Omega = (-2\pi i) t(k, k' : (\rho' + i0+)^2) (\rho'/2) \delta(\rho + \rho')$ and $t(k, k' : z)$ is Hölder continuous on $L_2(R^3)$. Let $K^{(j,k)} = (2\pi i)^{-2} \int \int ((-1)^j l_1 + i0+)^{-1} ((-1)^k l_2 - i0+)^{-1} K((\rho + l_1 + (-1)^{j+k} l_2) \hat{k}, (\rho' + l_2) \hat{k}') dl_1 dl_2, j, k = 0, 1, K_- = \sum_k (K^{(0,k)} + (K^{(1,k)})^*), K_+ = (K)^*, K = K_- + K_+$. If $(\rho^\beta \Omega) \in \mathbf{H} \otimes \mathbf{H}, \beta = 0, 1, 2, \|\Omega\| < 1$, then the Gelfand-Levitan equation $K + \Omega + (K_- \Omega)_- + (\Omega K_+)_+ = 0$ has a solution in $\mathbf{H} \otimes \mathbf{H}$ and $v(k - k') \in \mathbf{H} \otimes \mathbf{H}$ is given by $v(k - k') = (\rho - \rho') (\pi i)^{-1} \int K((\rho + 1) \hat{k}, \rho' + 1) \hat{k}' dl$.

(Received January 25, 2006)