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On the behavior of the minimal Riesz s -energy for large values of s .

For $A \subset \mathbf{R}^{d'}$ define

$$\mathcal{E}_s(A, N) := \inf_{\{x_1, \dots, x_N\} \subset A} \sum_{i \neq j} \frac{1}{|x_i - x_j|^s}, \quad s > 0.$$

Authors earlier showed that for a d -rectifiable set $A \subset \mathbf{R}^{d'}$ (Lipschitz image of a d -dimensional compact set, $d \leq d'$)

$$\lim_{N \rightarrow \infty} \frac{\mathcal{E}_s(A, N)}{N^{1+s/d}} = \frac{C_{s,d}}{\mathcal{H}_d(A)^{s/d}}, \quad s > d,$$

where $C_{s,d}$ is a positive (unknown for $d > 1$) constant independent of A , and \mathcal{H}_d is the d -dimensional Hausdorff measure. Let also

$$\delta_N(A) := \sup_{\{x_1, \dots, x_N\} \subset A} \min_{i \neq j} |x_i - x_j|$$

be the best-packing radius on A and

$$\Delta_d(A) := \lim_{N \rightarrow \infty} \delta_N(A) \cdot N^{1/d}.$$

We show the following:

1. For every $d > 1$

$$\lim_{s \rightarrow \infty} C_{s,d} = \frac{1}{\Delta_d([0, 1]^d)}.$$

2. If K is the classical Cantor subset of $[0, 1]$, then for s sufficiently large

$$0 < \liminf_{N \rightarrow \infty} \frac{\mathcal{E}_s(K, N)}{N^{1+s \log_2 3}} < \limsup_{N \rightarrow \infty} \frac{\mathcal{E}_s(K, N)}{N^{1+s \log_2 3}} < \infty.$$

3. If $A \subset \mathbf{R}^d$ is a d -rectifiable set, then

$$\Delta_d(A) = \mathcal{H}_d(A)^{1/d} \cdot \Delta_d([0, 1]^d).$$

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