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**Doron S Lubinsky\*** (lubinsky@math.gatech.edu), School of Mathematics, Georgia Institute of Technology, Atlanta, GA 30332-0160. *Which Weights on the real line admit  $L_p$  Jackson Theorems?*

In about 1910, S.N. Bernstein posed the problem of approximating by polynomials on the whole real line, with a weight to “kill” the growth of the polynomials at infinity. The problem was solved independently by Achieser, Mergelyan and Pollard in the 1950’s. For example, the polynomials are dense with weight  $\exp(-|x|^a)$  when  $a$  is at least 1, but not when  $a$  is less than 1.

Subsequent research focused on the degree of approximation: what rate can be achieved for functions with a given smoothness? The goal here was a weighted analogue of Jackson’s theorem for  $[-1, 1]$ .

In this paper we fix  $p$  between 1 and infinity inclusive, let  $q$  be its conjugate parameter, and let  $W$  be a continuous positive function on the real line. We characterize which  $W$  admit a Jackson theorem in the given  $L_p$ . The answer is surprisingly simple: we need  $\|W\|_{L_p(a, \text{infinity})} \|W^{-1}\|_{L_q(0, a)}$  to have limit 0 as  $a$  approaches infinity, with a similar limit at  $-\infty$ . (Received January 19, 2006)