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**David E. Edmunds, W. Desmond Evans and Georgi E. Karadzhov\***

(geremika@yahoo.com), Institute of Mathematics and Informatics, Bulgarian Academy of sciences, 8 G. Bonchev Street, 1113 Sofia, Bulgaria. *Sharp estimates of the embedding constants for Besov spaces.*

Sharp estimates are obtained for the rates of blow up of the norms of embeddings of Besov spaces in Lorentz spaces as the parameters approach critical values. More precisely, we consider the homogeneous Besov spaces  $b_{p,q}^s$  on the  $n$ -th dimensional Euclidean space, defined as the completion of all smooth functions with compact support with respect to the quasinorm  $(\int_0^\infty (t^{-s}\omega_p^k(t, f))^q dt/t)^{1/q}$ , where  $0 < s < k$ ,  $0 < p < \infty$ ,  $0 < q \leq \infty$ , and  $\omega_p^k(t, f)$  stands for the usual modulus of continuity of order  $k$  in the  $L^p$ - metric. A typical result is the following one:

Let  $1 < p < \infty$ ,  $1 \leq q \leq \infty$ ,  $0 < \sigma < 1$ ,  $1/r = (1 - \sigma)/p$ ,  $k = n/p$ . Then as  $\sigma \rightarrow 1$ ,

$$b_{p,q}^{k\sigma} \subset (1 - \sigma)^{1-1/p} L_{r,q}.$$

Here  $L_{r,q}$  is the Lorentz space and the above result is sharp.

Similar results are proved for the cases  $k > n/p > s$  and  $s > n/p$  as  $s$  approaches  $n/p$ . The main tools are real interpolation and the following pointwise estimate for the rearrangement:

Let  $1 \leq p < n/(k - 2)$  if  $k \geq 3$ , and  $1 \leq p \leq \infty$  if  $k = 1, 2$ . Then for all smooth functions with compact support,

$$f^{**}(t) - f^{**}(2t) \lesssim t^{-1/p} \omega_p^k(t^{1/n}, f).$$

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