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Yoshimi Saito* (saito@math.uab.edu), 1507 Wellington Rd, Birmingham, AL 35209. *Nonlinear eigenvalue problem for the Hardy operator.*

Let $Tf(x) = v(x) \int_a^x u(t)f(t)dt$ be the Hardy operator from $L^p(I)$ into $L^q(I)$, where $I = (a, b)$ is a finite interval, $u \in L^{p'}(I)$, $v \in L^q(I)$, $1 < p, q < \infty$, $p^{-1} + p'^{-1} = 1$ and $u(x)v(x) \neq 0$ a.e. on I .

The stationary vectors f are defined and the "eigenvalue" associated with f is given by $\mu_f = \|Tf\|_q / \|f\|_p$. The stationary vector f satisfies the equation

$$T^*((Tf)^{(q-1)}) = \lambda^{-1} f^{(p-1)}$$

where $g^{(t)} = |g|^{t-1}g$ (odd power function) and $\lambda = \lambda_f = \|Tf\|_q^q / \|f\|_p^p$.

We shall discuss the property of these eigenvalues.

In the case $p = q$ we can show how to find all stationary vectors and it turned out that these eigenvalues are the same as the approximate numbers of T . The Prüfer transform will be used as a useful tool.

In the general case, we can show the existence of the eigenvalues. But it seems that many other questions are still unanswered.

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