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Santiago Simanca* (santiago@math.unm.edu), Department of Mathematics and Statistics,
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Consider a closed Riemannian manifold (\tilde{M}, \tilde{g}) , and let (M, g) be an immersed submanifold endowed with the induced metric g from that on the ambient space. Let α be the second fundamental form, a symmetric two-tensor over M with values in the normal bundle. We consider the functional $\int_M \|\alpha\|^2 d\mu_g$ over the space of all immersions of M into the ambient space that represent a fixed homology class, and derive its Euler-Lagrange equation under isometric deformations. We describe basic properties of the critical points in codimensions one and two, respectively, and present examples. In particular, we show that a degree d curve in $\mathbb{C}\mathbb{P}^n$ is a critical point. We outline a program that intends to use this result to show that, among all embedded representatives of the homology class of this curve in $\mathbb{C}\mathbb{P}^2$, the complex representative achieves the lowest possible genus (this a theorem of Kronheimer-Mwroka whose only known proof uses the theory of Seiberg and Witten). (Received February 07, 2006)