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David A. Vogan* (dav@math.mit.edu), Room 2-281, MIT, 77 Massachusetts Avenue,
Cambridge, MA 02139. *Cohomological induction and irreducible representations.*

Let K be a maximal compact subgroup of a real reductive group G , with corresponding Cartan involution θ . Let $\mathfrak{q} = \mathfrak{l} + \mathfrak{u}$ be a θ -stable parabolic subalgebra of the complexified Lie algebra \mathfrak{g} , and L the normalizer of \mathfrak{q} in G . Zuckerman's cohomological induction begins with an irreducible $(\mathfrak{l}, L \cap K)$ -module Z and forms the "generalized Verma module" $V = U(\mathfrak{g}) \otimes_{\mathfrak{q}} Z$, which is a $(\mathfrak{g}, L \cap K)$ -module. Finally he applies to V the "(Bernstein)-Zuckerman functors" L_p , which carry $(\mathfrak{g}, L \cap K)$ -modules to (\mathfrak{g}, K) -modules. The "cohomologically induced representations" are $L_p(V)$.

In this setting, suppose J is any irreducible subquotient of the generalized Verma module V . I will show that $L_p(J)$ is always a direct sum of irreducible representations, which can be computed explicitly using the (proved) Kazhdan-Lusztig conjectures.

This work is motivated by Wai Ling Yee's recent calculation of the signatures of invariant Hermitian forms on J (in case L is abelian). The hope is that her work will in turn determine the signatures of induced forms on $L_p(J)$. (Received January 23, 2006)