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Superquadratic functions in several variables.

A function $f : K_m \rightarrow \mathbb{R}$ is superquadratic if for every $x \in K_m$ there exists a vector $c(x) \in \mathbb{R}^m$ such that $f(y) \geq f(x) + \langle c(x), y - x \rangle + f(|y - x|)$ holds for all $y \in K_m$ ($K_m = [0, \infty)^m$). Superquadraticity for non negative functions is stronger than convexity unless f takes negative values. For superquadratic functions we establish refinements of Jensen's inequality and its counterpart. Let $f : K_m \rightarrow \mathbb{R}$ be superquadratic and $f(0) = 0$. Let $p_i \in \mathbb{R}$, $p_i \geq 0$, $i = 1, \dots, n$, and $P_n = \sum_{i=1}^n p_i > 0$. Define $\bar{x} = (1/P_n) \sum_{i=1}^n p_i x_i$, $\bar{y} = (1/P_n) \sum_{i=1}^n p_i f(x_i)$. We prove that, for any $x \in K_m$, if $c(x)$ is as above and $a, b \in K_m$ are arbitrary vectors then

$$\begin{aligned} f(a) + \langle c(a), \bar{x} - a \rangle + \frac{1}{P_n} \sum_{i=1}^n p_i f(|x_i - a|) &\leq \bar{y} \\ &\leq f(b) + \frac{1}{P_n} \sum_{i=1}^n p_i \langle c(x_i), x_i - b \rangle - \frac{1}{P_n} \sum_{i=1}^n p_i f(|x_i - b|). \end{aligned}$$

Examples: 1) $f(x) = (\|x\|_p)^p$, $p \geq 2$ is superquadratic. 2) $f(x) = \|x\|^2 \ln \|x\|$ if $x \neq 0$, $f(0) = 0$ is superquadratic for $m = 1$ but not for $m = 2$. (Received February 05, 2006)