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Kevin Woods* (kwoods@math.berkeley.edu), Department of Mathematics, University of California, Berkeley, 970 Evans Hall, Berkeley, CA 94720-3840. *Periods of Ehrhart quasi-polynomials.*

Given a rational polytope P , $i_P(n) := \#(nP \cap \mathbb{Z}^d)$ is a *quasi-polynomial* in n , that is, there exist a period \mathcal{D} and polynomial functions $f_1, \dots, f_{\mathcal{D}}$ such that $i_P(n) = f_j(n)$ for $n \equiv j \pmod{\mathcal{D}}$. This function $i_P(n)$ is called the *Ehrhart quasi-polynomial* of P . The minimum period of $i_P(n)$ must divide $\mathcal{D}(P) = \min\{n \in \mathbb{Z}_{>0} : nP \text{ is an integral polytope}\}$, and for most polytopes the minimum period is exactly $\mathcal{D}(P)$. In some interesting real world — real math-world, at least — examples, the period of $i_P(n)$ is mysteriously smaller.

This motivates the study of periods of Ehrhart quasi-polynomials, and we will present a survey of some results along this line. We give examples to show that difference between \mathcal{D} and the actual minimal period can be arbitrarily “bad.” We also examine better bounds on the periods of the *coefficients* of the Ehrhart quasi-polynomials. Finally, we examine the computational complexity of determining the minimum period, using the tool of rational generating functions. Along the way, we will present a number of open questions related to these periods. (Received March 07, 2006)