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John K Hunter* (jkhunter@ucdavis.edu), Department of Mathematics, University of California at Davis, Davis, CA 95616. *Variational systems of nonlinear wave equations*. Preliminary report.

We consider systems of wave equations that are governed by variational principles whose Lagrangians are quadratic functions of the derivatives of the wave-field with coefficients depending on the wave-field itself. A specific example is a system of equations that describes the propagation of orientation waves in a massive liquid crystal director field. The vacuum Einstein equations of general relativity also belong to this class, after the imposition of a suitable gauge.

We introduce notions of genuine nonlinearity and linear degeneracy for variational wave equations that are analogous to, but different from, the corresponding notions for hyperbolic systems of conservation laws. We derive a new weakly nonlinear asymptotic equation for waves that lose genuine nonlinearity in a system, and use it to study the propagation of ‘twist’ waves in a massive director field. As we will show, the linearly degenerate ‘twist’ waves couple with genuinely nonlinear ‘splay’ waves, whose wave-amplitude satisfies the Hunter-Saxton equation. (Received February 27, 2006)