Graeme W. Milton* (milton@math.utah.edu), Department of Mathematics, University of Utah, Salt Lake City, UT 84101, and Hyeonbae Kang (hkang@math.snu.ac.kr). Proof of the conjectures of Polya-Szego and Eshelby.

It is well known that a sphere has the minimum perimeter for a given volume. Is the sphere optimal with respect to other properties, such as electrical properties? In 1951 Polya and Szego conjectured that the inclusion whose polarization tensor has minimal trace for a given volume would take the shape of a sphere. Here we prove their conjecture. We also prove, in three dimensions the weak form of a conjecture that Eshelby made in 1961. Specifically we show that an inclusion must be of ellipsoidal shape if for any uniform elastic loading the field inside the inclusion is uniform. In two dimensions it has previously been proved by Sendeckyj for elasticity, and Ru and Schiavone, for conductivity, that the strong form of the conjecture is true: the inclusion must be elliptical in shape if the field inside is uniform with respect to a single loading. (Received August 14, 2006)