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Yury Grabovsky* (yury@temple.edu), Temple University 038-16, Wachman Hall, Rm 638, 1805 N. Broad St., Philadelphia, PA 19122-6094, and **Tadele Mengesha** (mengesha@temple.edu), Temple University 038-16, Wachman Hall, Rm 638, 1805 N. Broad St., Philadelphia, PA 19122-6094. *Quasiconvexity-based sufficient conditions for strong local minima: the smooth case.*

We present the multi-dimensional analog of the Weierstrass sufficiency theorem in Calculus of Variations for C^1 extremals. The sufficient conditions feature the expected naturally strengthened versions of known necessary conditions. Additionally, to limit the technical complications, we impose some regularity and growth conditions at infinity. To prove the sufficiency theorem, we adapt the tools developed for the “Direct Method in Calculus of Variations” to suit our goals. The key idea is due to L. C. Young: instead of studying what happens to a given Lagrangian under different variations, we examine the action of a given variation on a whole class of Lagrangians. The key tool is the Decomposition Lemma, first proved by Jan Kristensen, that allows us to split the action of the variation into the weak part and the strong part, acting independently. Positivity of second variation ensures that the weak part cannot decrease the functional, while the quasiconvexity conditions ensure that the strong part is unable to decrease the functional either. (Received July 10, 2006)