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We investigate the line arrangement that results from intersecting d complete flags in \mathbb{C}^n . We give a combinatorial description of the matroid $T_{n,d}$ that keeps track of the linear dependence relations among these lines.

We prove that the bases of the matroid $T_{n,3}$ characterize the triangles with holes which can be tiled with unit rhombi. More generally, we provide evidence for a conjectural connection between the matroid $T_{n,d}$, the triangulations of the product of simplices $\Delta_{n-1} \times \Delta_{d-1}$, and the arrangements of d tropical hyperplanes in tropical $(n-1)$ -space.

Our work provides a simple and effective criterion to ensure the vanishing of many Schubert structure constants in the flag manifold, and a new perspective on Billey and Vakil's method for computing the non-vanishing ones. (Received August 15, 2006)