

1020-06-114

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We first consider the linear algebra of all matrices with real entries of some fixed order  $n$ . We use  $I$  to denote the identity matrix. We partially order these matrices entrywise to obtain a partially ordered linear algebra (POLA). We next select an idempotent matrix  $E$  ( $E^2 = E$ ) and a nilpotent matrix  $M$  ( $M^2 = 0$ ). Is it possible that  $2I \leq E + M$ ? No, this inequality is not possible in this POLA.

We next consider the infinite matrix algebra of column-finite matrices with real entries. As above, we partially order these matrices entrywise to get a POLA. In this case it is possible to find idempotent  $E$  and nilpotent  $M$  such that  $2I \leq E + M$ . We refer to this latter inequality as a strange inequality.

If we consider four types of matrices: idempotent  $E^2 = E$ , nilpotent  $M^2 = 0$ , involution  $S^2 = I$  and imaginary  $J^2 = -I$ , then there are ten basic strange inequalities involving these types of matrices. All of these inequalities can be obtained in the POLA of column-finite matrices.

Sample result 1. If  $I \leq S + J$ , then  $S$  and  $J$  cannot commute.

Sample result 2. If  $3I \leq E + S$ , then  $E$  and  $S$  cannot be nonnegative matrices.

Many other results will be discussed. (Received August 21, 2006)