

1021-05-28

François Bergeron and **Aaron Lauve*** (lauve@lacim.ugam.ca), LaCIM, University of Quebec at Montreal, Case Postale 8888, succursale Centre-ville, Montreal, Quebec H3C 3P8, Canada.

Noncommutative invariants and coinvariants of the symmetric group. Preliminary report.

The algebras NCSym_n and Sym_n ($n \in \mathbb{N}_+$) are defined to be the \mathfrak{S}_n -invariants inside $\mathbb{Q}\langle A_n \rangle$ (resp. $\mathbb{Q}[X_n]$), the polynomial functions on a noncommutative alphabet A_n (resp. commutative, X_n) of cardinality n . The abelianization ($a_i \mapsto x_i$) realizes Sym_n as a quotient of NCSym_n . Here, we view it as a subspace. We realize Sym_n as the \mathfrak{S}_n -invariants inside NCSym_n for a second, natural action of the symmetric group on NCSym_n and describe the coinvariants explicitly. Some surprising identities on the ordinary generating function for the Bell numbers appear as an immediate corollary. In case $n = \infty$, we obtain new information on the (Hopf) algebraic structure of NCSym_n . A new POset structure on the set partitions with common, underlying integer partition plays a role here.

Time permitting, we outline similar results for Hivert's r - QSym_n algebras ($r, n \in \mathbb{N}_+ \cup \{\infty\}$) and their noncommutative analogues. The algebra Sym_n and Gessel's quasisymmetric functions appear at the extremal values of r . (Received July 11, 2006)