

1021-35-209

Michael Plum* (michael.plum@math.uni-karlsruhe.de), University of Karlsruhe, Faculty of Mathematics, Institute for Analysis, Englerstrasse 2, D-76128 Karlsruhe, Germany.

Computer-assisted Proofs for Nonlinear Elliptic Boundary Value Problems.

The lecture is concerned with numerical enclosure methods for nonlinear elliptic boundary value problems. Here, analytical and numerical methods are combined to prove rigorously the existence of a solution in some "close" neighborhood of an approximate solution computed by numerical means. Thus, besides the existence proof, verified bounds for the error (i.e. the difference between exact and approximate solution) are provided.

For the first step, consisting of the computation of an approximate solution ω in some appropriate Sobolev space, no error control is needed, so a wide range of numerical methods (including multigrid schemes) is at hand here. Using ω , the given problem is rewritten as a *fixed-point equation* for the error, and the goal is to apply a *fixed-point theorem* providing the desired error bound.

The conditions required by the chosen fixed-point theorem are now verified by a combination of analytical arguments (e.g. explicit Sobolev embeddings) and verified numerical computations of certain auxiliary terms.

The method is illustrated by several examples (on bounded as well as on unbounded domains), where in particular it gives existence proofs in cases where no purely analytical proof is known.

(Received September 05, 2006)