Felix Lazebnik* (lazebnik@math.udel.edu), Department of Mathematical Sciences, University of Delaware, Newark, DE 19716, Oleg Pikhurko (pikhurko@cmu.edu), Department of Mathematical Sciences, Carnegie Mellon University, Pittsburgh, PA 15213, and Andrew J. Woldar (andrew.woldar@villanova.edu), Department of Mathematical Sciences, Villanova University, Villanova, PA 10085. Maximum Number of Colorings of \((2k,k^2)\)-Graphs.

Let \(F_{2k,k^2}\) consist of all simple graphs on \(2k\) vertices and \(k^2\) edges. For a simple graph \(G\) and a positive integer \(\lambda\), let \(P_G(\lambda)\) denote the number of proper vertex colorings of \(G\) in at most \(\lambda\) colors, and let \(f(2k,k^2,\lambda) = \max\{P_G(\lambda) : G \in F_{2k,k^2}\}\).

Let \(K_{k,k}\) denote the complete bipartite graph with each partition having \(k\) vertices. We prove that \(f(2k,k^2,3) = P_{K_{k,k}}(3)\) and \(K_{k,k}\) is the only extremal graph. We also prove that \(f(2k,k^2,4) = (6 + o(1))4^k\) as \(k \to \infty\). (Received September 10, 2006)