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Paul Pedersen* (pedersen@lanl.gov), Mail Stop B265, Los Alamos National Laboratory, Los Alamos, NM 87545. *Partial Differential Equations and Real, Finitely Generated, Commutative and Associative Algebras.*

A real, finitely generated, commutative, and associative algebra (hereafter Algebra) is a homomorphic image of $R[y] = R[y_1, \dots, y_n]$ (the Algebra of polynomials in n variables over the reals). The kernel of this homomorphism is an ideal in $R[y]$. Given any polynomial $P(y)$ in $R[y]$ we show how to form an Algebra having as its kernel the ideal $\langle P(y) \rangle$. We write this Algebra as $R[\alpha] = R[\alpha_1, \dots, \alpha_n]$ where α_i is the homomorphic image of y_i . We discuss the fact that the expression $e^{\sum x_i \alpha_i}$ can be used to find a basis for all analytic solutions to $P(D_1, \dots, D_n)u(x_1, \dots, x_n) = 0$ where D_i is the partial derivative with respect to x_i . So, for example, we can use this method to find a basis for analytic solutions to the heat equation, Laplace's equation, or the wave equation in any number of variables. In the special case where the ideal is $\langle y^2 + 1 \rangle$ the Algebra is the complex numbers and we get that functions of a complex variable can be used to find all analytic solutions to Laplace's equation in 2 variables. In the special case of Laplace's equation in n variables we can use this method to find a simple recursion relating degree m and degree $m+1$ spherical harmonics. (Received September 07, 2006)