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Daniele Morbidelli* (morbidel@dm.unibo.it), Dipartimento di Matematica, Università di Bologna, Piazza di Porta S. Donato, 5, 40127 Bologna, Italy. *The Liouville theorem for conformal maps in diagonal metrics of Grushin-type.*

We consider conformal maps with respect to the control distance d generated by the following family of vector fields in $\mathbb{R}^p \times \mathbb{R}^q$:

$$X_j = \frac{\partial}{\partial x_j}, \quad Y_\lambda = (\alpha + 1)|x|^\alpha \frac{\partial}{\partial y_\lambda}, \quad j = 1, \dots, p, \quad \lambda = 1, \dots, q.$$

Here $\alpha > 0$ is a fixed parameter.

It can be checked that all maps of the form $(x, y) \mapsto (Ax, By + b)$, where $A \in O(p)$, $B \in O(q)$ and $b \in \mathbb{R}^q$ are isometries. Moreover, all anisotropic dilations of the form $(x, y) \mapsto \delta_t(x, y) := (tx, t^{\alpha+1}y)$, where $t > 0$, are conformal too. More examples of conformal maps can be constructed by means of the following inversions:

$$(x, y) := z \mapsto \delta_{\|z\|^{-2}} z,$$

where $\|z\| = \{|x|^{2(\alpha+1)} + |y|^2\}^{1/(2(\alpha+1))}$.

Our main result states that if $p \geq 3$, $q \geq 1$ and f is a conformal map between connected sets of $\mathbb{R}^p \times \mathbb{R}^q$, then f can be written as a composition of the mentioned isometries, dilations and inversions. (Received September 08, 2006)