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*Color critical hypergraphs and forbidden configurations.*

Lovász proved that

$$|\mathcal{E}| \leq \binom{n}{k-1}$$

for a 3-color critical  $k$ -uniform hypergraph. We prove

**Theorem 1.** *Let  $\mathcal{E} \subseteq \binom{[m]}{k}$  be a  $k$ -uniform set system on  $|X| = m$ . Let us fix an ordering  $E_1, E_2, \dots, E_t$  of  $\mathcal{E}$  and a prescribed partition  $A_i \cup B_i = E_i$  ( $A_i \cap B_i = \emptyset$ ) for each member of  $\mathcal{E}$ . Assume that for all  $i = 1, 2, \dots, t$  there exists a partition  $C_i \cup D_i = X$  ( $C_i \cap D_i = \emptyset$ ), such that  $E_i \cap C_i = A_i$  and  $E_i \cap D_i = B_i$ , but  $E_j \cap C_i \neq A_j$  and  $E_j \cap D_i \neq B_j$  for all  $j < i$ . Then*

$$t \leq \binom{m}{k-1} + \binom{m}{k-2} + \dots + \binom{m}{0}$$

Theorem 1 was motivated by the following. Let  $F$  be a  $k \times l$  0-1 matrix, then  $\text{forb}(m, F)$  is the maximum  $n$  such that there exists an  $m \times n$  simple matrix  $A$  such that no column and/or row permutation of  $F$  is a submatrix of  $A$ . Let  $K_k$  denote the full  $k \times 2^k$  simple 0-1 matrix.

**Theorem 2.** *Let  $F$  be contained in  $F_B = [K_k | t \cdot (K_k - B)]$  for an  $k \times (k+1)$  matrix  $B$  consisting of one column of each possible column sum. Then  $\text{forb}(m, F) = O(m^{k-1})$ .*

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