

1024-05-208

R Anstee, Vancouver, **B Fleming**, Vancouver, **Z Furedi*** (z-furedi@math.uiuc.edu), 1409 W Green Str, Urbana, IL 61801, and **A Sali**, Budapest, Hungary. *Color critical hypergraphs and forbidden configurations.*

A k -uniform hypergraph (V, \mathcal{E}) is 3-color critical if it is not 2-colorable, but for all $E \in \mathcal{E}$ the hypergraph $(V, \mathcal{E} \setminus \{E\})$ is 2-colorable. Lovász proved in 1976, that

$$|\mathcal{E}| \leq \binom{n}{k-1}$$

for a 3-color critical k -uniform hypergraph. Here we prove the following generalization.

Let $\mathcal{E} \subseteq \binom{[m]}{k}$ be a k -uniform set system on an underlying set X of m elements. Let us fix an ordering E_1, E_2, \dots, E_t of \mathcal{E} and a prescribed partition $A_i \cup B_i = E_i$ ($A_i \cap B_i = \emptyset$) for each member of \mathcal{E} . Assume that for all $i = 1, 2, \dots, t$ there exists a partition $C_i \cup D_i = X$ ($C_i \cap D_i = \emptyset$), such that $E_i \cap C_i = A_i$ and $E_i \cap D_i = B_i$, but $E_j \cap C_i \neq A_j$ and $E_j \cap C_i \neq B_j$ for all $j < i$. (That is, the i th partition cuts the i th set as it is prescribed, but does not cut any earlier set properly.) Then

$$t \leq \binom{m}{k-1} + \binom{m}{k-2} + \dots + \binom{m}{0}.$$

This leads to a sharpening of Sauer's bound for $\text{forb}(m, F)$, where F is a $k \times l$ 0-1 matrix. (Received January 09, 2007)