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Peter Vamos and **Sylvia M Wiegand*** (swiegand@math.unl.edu), Mathematics Department, University of Nebraska, Lincoln, NE 68588-0130. *Sums of 2 units and almost diagonal matrices*. Preliminary report.

We consider two questions and relations between them:

Question 1 For which rings is every element a sum of two units of the ring?

Question 2 For which rings is every matrix equivalent to an “almost diagonal” matrix? (“almost diagonal” is explained later.)

Regarding Question 1, the ring of integers \mathbb{Z} is not an example. For every principal ideal domain R , however, the ring of $n \times n$ matrices over R is an example, if $n \geq 2$. In fact $n \times n$ matrices over *elementary divisor rings* are diagonalizable, and from this it follows that, if $n \geq 2$, then every matrix is a sum of two units of the ring.

We show that, for $n \geq 2k$ and R a commutative ring, if a matrix $X \in M_n(R)$ is k -banded (that is, for every i, j with $|i - j| \geq k$, the (i, j) -entry of X is 0, then $X = U_1 + U_2$ with U_1, U_2 invertible. Thus, if R satisfies $\forall X \in M_n(R), S \sim B$, for some k -banded matrix B , then every $X \in M_n(R)$ has $X = U_1 + U_2$ for some invertible matrices U_1, U_2 .

We also show that certain Prüfer domains satisfy the k -banded property above. (Received January 04, 2007)