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**Jay Shapiro\*** ([jshapiro@gmu.edu](mailto:jshapiro@gmu.edu)), Department of Mathematics, George Mason University,  
Fairfax, VA 22030. *Irreducibility in the total ring of quotients.*

Let  $R$  be a ring whose total ring of quotients  $Q$  is von Neumann regular. Generalizing the work in [L. Fuchs, W. Heinzer, and B. Olberding, Commutative ideal theory without finiteness conditions: irreducibility in the quotient field, Abelian groups, rings, modules, and homological algebra, 121–145, Lect. Notes Pure Appl. Math., 249], we investigate the structure of  $R$  when it admits an ideal that is irreducible as a submodule of the total ring of quotients. In particular, we characterize those rings which contain a maximal ideal that is irreducible in  $Q$  the total ring of quotients of  $R$ . It has been shown that an integral domain has a  $Q$ -irreducible ideal which is a maximal ideal if and only if  $R$  is a valuation domain. We show that when the total ring of quotients of  $R$  is von Neumann regular, then having a maximal ideal that is  $Q$ -irreducible is equivalent to a valuation like property. This property in turn is equivalent to, among other things,  $R$  being a pullback of the form  $Q \times_{Q/P} V$ , where  $Q$  is a von Neumann regular ring,  $P \in \text{Spec}(Q)$  and  $Q/P$  is the quotient field of the valuation ring  $V$ . (Received December 15, 2006)