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University of Maryland Eastern Shore, Princess Anne, MD 21853. *Differential Geometric  
Properties of Brownian Motion in the Neighborhood of a Submanifold.*

Let  $M$  be a complete connected smooth  $n$ -dimensional Riemannian manifold and let  $P$  be a  $q$ -dimensional smooth submanifold smoothly embedded in  $M$ . Let  $L = (1/2)A + b + c$  be a second order differential operator where  $A$  is the Laplace-Beltrami operator on  $M$ ,  $b$  a smooth vector field on  $M$  and  $c$  a smooth potential term on  $M$ . Let  $D$  be a tubular neighborhood of  $P$  in  $M$  and let  $p(t,D,x,y)$  denote the Dirichlet heat kernel of  $D$  relative to the operator  $L$ . Using Fermi coordinates (which generalize normal coordinates), we will show here that the integral of  $p(t,D,x,y)$  over the submanifold  $P$  (with respect to the Riemannian measure  $dy$ ) generalizes the Dirichlet heat kernel  $p(t,D,x,y)$ . As a consequence the expansion of the integral in powers of  $t$  generalizes the usual Minakshisundaram-Pleijel heat kernel expansion of the Dirichlet heat kernel. In this paper we will specialize to the case that the submanifold  $P$  is a hypersurface. We will show that in this case, the leading coefficients of the expansion, which are local geometric invariants, are easily computable (at the center of Fermi coordinates) in terms of the curvature of  $M$ , the curvature of the submanifold  $P$ , the second fundamental form of  $P$  and a "torsion" operator. (Received November 02, 2006)