1025-03-197 **Teruyuki Yorioka*** (styorio@ipc.shizuoka.ac.jp), Ohya 836, Shizuoka, 422-8029, Japan.
$\mathbb{P}_{max}$ variations and ccc structures.

$\mathbb{P}_{max}$ was introduced by W. Hugh Woodin. This is a strong tool to deal with structures of size $\aleph_1$, e.g. Suslin trees. In fact, Larson, Shelah–Zapletal, and Woodin himself (and so on) have investigated several structures of size $\aleph_1$ in extensions by suitable $\mathbb{P}_{max}$ variations. The main observation of these studies is a $\mathbb{P}_{max}$-iteration of a iterable pair.

We talk on constructions of $\mathbb{P}_{max}$-iterations keeping or destroying given ccc structures of size $\aleph_1$. The following is one of our constructions: Suppose $\diamondsuit$, and that (M, I) is an iterable pair, $\mathbb{P}$ and $\mathbb{Q}$ are member of M which are ccc partial order of size $\aleph_1$ in M such that $\mathbb{P} \times a(\mathbb{Q})$ is also ccc in M, where $a(\mathbb{Q})$ is a p.o. adding an antichain by finite approximations. Then there exists an iteration j of (M, I) of length ω_1 such that $j(\mathbb{P})$ is still ccc and there exists a $(j[M], a(j(\mathbb{Q})))$-generic filter.

This study may introduce interesting consistency results by some suitable $\mathbb{P}_{max}$ variations. We explain one application of $\mathbb{P}_{max}$ variations of destructible gaps, which is an analogy of the result due to Todorčević. (Received January 22, 2007)