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Given a distribution of pebbles on the vertices of a graph G , a *pebbling move* takes two pebbles from one vertex and puts one on a neighboring vertex. The *pebbling number* $\Pi(G)$ is the least k such that for every distribution of k pebbles and every vertex r , a pebble can be moved to r . The *optimal pebbling number* $\Pi_{OPT}(G)$ is the least k such that some distribution of k pebbles permits reaching each vertex.

Using new tools (“Squishing” and “Smoothing” Lemmas), we give short proofs of prior results on these parameters for paths, cycles, trees, and hypercubes, a new linear-time algorithm for computing $\Pi(G)$ on trees, and new results on $\Pi_{OPT}(G)$. If G is connected and has n vertices, then $\Pi_{OPT}(G) \leq \lceil 2n/3 \rceil$ (tight for paths and cycles). Let $a_{n,k}$ be the largest $\Pi_{OPT}(G)$ over n -vertex connected graphs with $\delta(G) \geq k$. Always $2(n-k)/(k+1) \leq a_{n,k} \leq 4n/(k+1)$, with a better lower bound when 3 divides k . Better upper bounds hold for n -vertex graphs with minimum degree k having large girth; a special case is $\Pi_{OPT}(G) \leq 16n/(k^2+17)$ when G has girth at least 5 and $k \geq 4$. Finally, we compute $\Pi_{OPT}(G)$ in special families such as prisms and Möbius ladders. (Received January 21, 2007)