1025-46-117 **Guillermo P. Curbera*** (`curbera@us.es`), Facultad de Matemáticas, Universidad de Sevilla, Aptdo. 1160, 41080 Sevilla, Sevilla, Spain. *Sobolev spaces and vector measures?*

We discuss the problem of refining the classical Sobolev inequality, here $\Omega \subset \mathbb{R}^n$ is a bounded open set (of measure one),:

$$\|u\|_{L^q(\Omega)} \leq C \| \, |\nabla u| \, \|_{L^p(\Omega)}, \quad q := \frac{np}{n-p},$$

by means of determining the largest Sobolev space to which the corresponding Sobolev imbedding

$$W_0^{i,p}(\Omega) \longrightarrow L^q(\Omega)$$

can be extended, for a fixed range space. This problem leads naturally to the consideration of Sobolev spaces for norms more general than the classical L^p norms, namely, for *rearrangement invariant norms*.

This has shown to be connected with the study of an associate kernel operator in one variable

$$Tf(t) := \int_0^1 f(s) s^{\frac{1}{n}-1} \, ds, \quad t \in [0,1].$$

The study of the optimal domains for this kernel operator can be done via the theory of **integration of scalar functions with respect to a vector measure**.

These tools allow to answer questions related to the optimal extension of the Sobolev imbedding, the compactness of the extension, and the possibility of further extensions beyond the optimal one. (Received January 19, 2007)