

1026-05-143

Antonio Laface, Gregory G. Smith and Mauricio Velasco* (velasco@math.cornell.edu),
112 Malott Hall, Cornell University, Ithaca, NY. *Picard-graded Betti numbers and Cox rings.*

The Cox ring of an algebraic variety X fits in the following analogy: $\text{Cox}(X)$ is to X as the bigraded ring of polynomials $k[a, b, c, d]$ is to $\mathbb{P}^1 \times \mathbb{P}^1$.

There is a large class of varieties, the so called Mori Dream Spaces, whose Cox rings are finitely generated algebras, that is, $\text{Cox}(X) = S/I$ for a homogeneous ideal I in a $\text{Pic}(X)$ -graded polynomial ring S .

The question of describing the ideal I and of understanding how it relates with the geometry of the variety is a fundamentally open problem.

In this talk we introduce a tool to investigate this question. We define complexes of vector spaces whose homology determines the $\text{Pic}(X)$ -graded Betti numbers of $\text{Cox}(X)$ and we show that these complexes can be studied with the methods of complex algebraic geometry (i.e. via Riemann-Roch and the Kawamata-Viehweg vanishing theorem).

As an application of this technique we give a new proof of the fact that the Cox rings of Del Pezzo surfaces (of degree > 1) are quadratic algebras. (Received February 23, 2007)