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**Alex V Kontorovich\*** ([alexk@math.columbia.edu](mailto:alexk@math.columbia.edu)), 2990 Broadway, MC 4406, Dept of Math, Columbia University, New York, NY 10027. *Hyperbolic Lattice Point Count in Infinite Volume with Applications to Sieves.*

There are very few examples of thin sets known to contain primes. Some of the most famous are the Piatetski-Shapiro prime number theorem and Friedlander and Iwaniec's polynomial  $X^2 + Y^4$  (and subsequently, Heath-Brown's polynomial  $X^3 + 2Y^3$ ). Consider Fermat's original problem of primes in the sum of two squares,  $c^2 + d^2$ , but take  $(c, d)$  to be the bottom rows of matrices in an infinite index non-elementary subgroup of  $SL(2, \mathbb{Z})$ . The work of Bourgain, Gamburd, and Sarnak implies that this set contains infinitely many "R-almost primes" (integers with at most R factors), but their theorem is so general that it gives an unspecified R. We will first execute a hyperbolic lattice point count in infinite volume to show that this set is indeed thin. Then we will use knowledge of an infinite volume spectral gap (expander property) to count this set along arithmetic progressions. Finally, we will use a combinatorial sieve to show that the set contains infinitely many R-almost primes, where R decreases as the Hausdorff dimension of the limit set of the subgroup approaches 1. (Received January 25, 2007)