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**Peter Keevash\*** ([keevash@caltech.edu](mailto:keevash@caltech.edu)), Mathematics, Caltech, Pasadena, CA 91125, and  
**Dhruv Mubayi.** *Set systems without a simplex or a cluster.*

A  $d$ -simplex is a collection of  $d+1$  sets with empty intersection, every  $d$  of which have nonempty intersection. A  $k$ -uniform  $d$ -cluster is a collection of  $d+1$  sets of size  $k$  with empty intersection and union of size at most  $2k$ .

We prove the following result which partially settles an old conjecture of Chvátal and a recent conjecture of Mubayi. For  $d \geq 2$  any  $\zeta > 0$  there is a number  $T$  such that the following holds for sufficiently large  $n$ . Let  $G$  be a  $k$ -uniform set system on  $[n] = \{1, \dots, n\}$  with  $\zeta n < k < n/2 - T$ , and suppose either that  $G$  contains no  $d$ -simplex or that  $G$  contains no  $d$ -cluster. Then  $|G| \leq \binom{n-1}{k-1}$  with equality only for the family of all  $k$ -sets containing a specific element.

In the non-uniform setting we obtain the following exact result that generalises a theorem of Milner, who proved the case  $d = 2$ . Suppose  $d \geq 2$  and  $G$  is a set system on  $[n]$  that does not contain a  $d$ -simplex, with  $n$  sufficiently large. Then  $|G| \leq 2^{n-1} + \sum_{i=0}^{d-1} \binom{n-1}{i}$ , with equality only for the family consisting of all sets that either contain some specific element or have size at most  $d-1$ .

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