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Dhruv Mubayi. *Set systems without a simplex or a cluster.*

A d -simplex is a collection of $d+1$ sets with empty intersection, every d of which have nonempty intersection. A k -uniform d -cluster is a collection of $d+1$ sets of size k with empty intersection and union of size at most $2k$.

We prove the following result which partially settles an old conjecture of Chvátal and a recent conjecture of Mubayi. For $d \geq 2$ any $\zeta > 0$ there is a number T such that the following holds for sufficiently large n . Let G be a k -uniform set system on $[n] = \{1, \dots, n\}$ with $\zeta n < k < n/2 - T$, and suppose either that G contains no d -simplex or that G contains no d -cluster. Then $|G| \leq \binom{n-1}{k-1}$ with equality only for the family of all k -sets containing a specific element.

In the non-uniform setting we obtain the following exact result that generalises a theorem of Milner, who proved the case $d = 2$. Suppose $d \geq 2$ and G is a set system on $[n]$ that does not contain a d -simplex, with n sufficiently large. Then $|G| \leq 2^{n-1} + \sum_{i=0}^{d-1} \binom{n-1}{i}$, with equality only for the family consisting of all sets that either contain some specific element or have size at most $d-1$.

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