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**Nick Rogers\*** (nfrogers@math.arizona.edu), Department of Mathematics, University of Arizona, 617 N. Santa Rita Av.e, Tucson, AZ 85721. *Descent on the Congruent Number Elliptic Curves.*

A positive integer  $n$  is said to be *congruent* if it is the area of a right triangle with rational sides. The problem of determining whether or not a particular number is congruent, and to produce a rational right triangle with the given area in the case that it is, dates to the ancient Greeks. An elementary argument shows that  $n$  is congruent whenever the corresponding “congruent number elliptic curve”  $y^2 = x^3 - n^2x$  has nonzero rank.

In 1983, Tunnell computed the central  $L$ -values of these elliptic curves, and deduced a criterion for the congruence of  $n$ . However, Tunnell’s result has a few drawbacks; perhaps most importantly, one direction depends on a weak form of the Birch and Swinnerton-Dyer conjecture, and is therefore non-constructive.

In this talk we’ll describe methods for obtaining upper bounds on the ranks of the congruent number elliptic curves via descent. Descent can be quite practical even for large  $n$ , and still provides the best known method for finding points on elliptic curves of rank at least 2. Various descents on the congruent number elliptic curves will be presented, along with applications to the average rank, the average size of the Tate-Shafarevich group, and the search for quadratic twists of large rank. (Received February 27, 2007)