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**Alex Postnikov, David Speyer and Lauren Williams\*** (lauren@math.harvard.edu), Dept. of Mathematics, Harvard University, 1 Oxford Street, Cambridge, MA 02138. *Matching polytopes, toric geometry, and the nonnegative part of the Grassmannian.*

In this paper we use toric geometry to investigate the topology of the totally non-negative part of the Grassmannian  $(Gr_{kn})_{\geq 0}$ .  $(Gr_{kn})_{\geq 0}$  is a cell complex whose cells  $\Delta_G$  can be parameterized in terms of the combinatorics of bicolored planar graphs  $G$ . To each cell  $\Delta_G$  we associate a complete fan  $F_G$  which is normal to a certain polytope  $P(G)$ . The combinatorial structure of the polytopes  $P(G)$  is reminiscent of the well-known Birkhoff polytopes, and we describe their face lattices in terms of matchings and unions of matchings of  $G$ . We also demonstrate a close connection between the polytopes  $P(G)$  and matroid polytopes. We then use the data of  $F_G$  and  $P(G)$  to define an associated toric variety  $X_G$ . We use our technology to prove that the cell decomposition of  $(Gr_{kn})_{\geq 0}$  is a CW complex, and furthermore, that the Euler characteristic of the closure of each cell of  $(Gr_{kn})_{\geq 0}$  is 1. (Received August 01, 2007)