1030-05-387 **Shu-Chung Liu*** (liularry_tw@yahoo.com.tw), Department of Applied Mathematics, National Hsinchu University of Education, Hsinchu City, 300, Taiwan, and **Sen-Peng Eu** and **Yeong-Nan Yeh**. *On the Congruences of Combinatorial Numbers.* Preliminary report.

The studies on the congruence of combinatorial numbers can be traced back to the famous and also age-old Pascal's Fractal, which has infinite triangles formed by the parities of binomial coefficients $\binom{n}{k}$. The problems on the congruences of combinatorial numbers modulo $p^k$ were considered for a long time, but just few results came out. Our work studies the congruences of Catalan numbers $C_n := \frac{1}{n+1}\binom{2n}{n}$ and Motzkin numbers $M_n = \sum_k \binom{n}{2k} C_k$ modulo 4 and 8. In particular, we justify the conjecture proposed by Deutsch and Sagan that $M_n \equiv_4 0$ if and only if $n = (4i+1)4^{j+1} - 1$ or $(4i+3)4^{j+1} - 2$ for $i, j \in \mathbb{N}$, and no $M_n$ is a multiple of 8. For the modularity of higher powers, $\mathbb{Z}_n^*$, the group of units of $\mathbb{Z}_n$, plays an important role. It is known that $\mathbb{Z}_{2^k}^*$ is isomorphic to $C_2 \oplus C_{2^{k-2}}$ $(k \geq 2)$, and the group $\mathbb{Z}_{p^k}^*$ is cyclic for any odd prime $p$. By transforming $\mathbb{Z}_n^*$ to an additive group, we develop more efficient formulas for enumerating the congruences of combinatoric numbers. Besides computing the congruences modulo higher powers, we use these formulas to show new proofs for Kummer's and Lucas' Theorems. (Received August 07, 2007)