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Aleksandar Jurisic* (ajurisic@valjhun.fmf.uni-lj.si), Faculty of Computer and Information Science, Trzaska 25, 1000 Ljubljana, Slovenia. *A characterization of Q-polynomial triangle-free distance-regular graphs with an eigenvalue multiplicity equal to the valency.*

Let Γ be a triangle-free distance-regular graph with valency $k \geq 3$ and diameter $d \geq 4$. The well known Terwilliger Tree Bound implies that the multiplicity of an eigenvalue is either 1 or at least k , and if equality holds, then the girth is ≤ 5 .

Let us assume Γ has an eigenvalue θ with multiplicity k . An important class of such examples comes from distance-regular graphs whose association scheme determined by its distance matrices is formally self-dual. Many interesting properties of such graphs have already been established, for example a parametrization with $d + 1$ parameters, namely with the cosine sequence $\omega_0, \omega_1, \dots, \omega_d$ corresponding to the eigenvalue θ .

Our main result is a characterization of the Q-polynomial property. Let us assume $1 \neq \omega_h$ ($1 \leq h \leq d$) and $\omega_{h+1} \neq \omega_{h-1} \neq \omega_h \neq \omega_{h+1}$ ($1 \leq h \leq d - 1$). We show that Γ is Q-polynomial with respect to the primitive idempotent E corresponding to θ if and only if

$$(\omega_1 - \omega_{i-1})(\omega_1 - \omega_{i+1}) = (\omega_2 - \omega_i)(1 - \omega_i), \quad \text{for all } i \in \{3, \dots, d - 1\},$$

or equivalently,

$$(\omega_1 - \omega_2)(\omega_1 - \omega_4) = (\omega_2 - \omega_3)(1 - \omega_3).$$

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