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Robert S. Maier* (rsm@math.arizona.edu), Mathematics Department, University of Arizona, Tucson, AZ 85721. *Transforming and reducing D-finite combinatorial sequences*. Preliminary report.

The solution $a := (a_n, n \in \mathbf{N})$ of a linear recurrence over \mathbf{C} with polynomial coefficients, and its formal generating function $F_a := \sum_{n \in \mathbf{N}} a_n z^n$, are said to be D-finite. The case of an order-1 recurrence, when F_a is hypergeometric, is nicest. We explore the algorithmic reduction of an order-(>1) recurrence to one of order 1, by manipulating the ODE satisfied by F_a . In particular, we consider reductions $F_a \rightarrow F_b$ of the form $F_a(z) = [1 - zQ(z)]^{-\theta} F_b(zP(z)/(1 - zQ(z)))$, with $P, Q \in \mathbf{C}[z]$, $\theta \in \mathbf{C}$. When a is a Heun sequence (one satisfying an order-2 recurrence with quadratic coefficients), we partially characterize when it may be so reduced. The characterization uses a classification of certain multisheeted covers of the Riemann sphere $\mathbf{P}^1(\mathbf{C})$ by itself. We give examples of nontrivial reductions, arising from the classical theory of automorphic functions. For instance, the Franel sequence $a_n = \sum_{k=0}^n \binom{n}{k}^3$, D-finite and of Heun type, can be reduced to a hypergeometric sequence $b_n, n \in \mathbf{N}$ in more than one way, due to relations among the modular groups $\Gamma_0(6), \Gamma_0(3), \Gamma_0(2), \Gamma(1)$. (Received August 07, 2007)