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Yufang Hao*, Department of Applied Mathematics, University of Waterloo, 200 University Ave. East, Waterloo, Ontario N2L3G1, Canada. *Applications of the Theory of Self-Adjoint Extensions of Symmetric Operator in Sampling and Interpolation.*

The Shannon classical sampling theorem, which is at the heart of information theory, states that a continuous B -bandlimited function $f(t)$ can be reconstructed from its discrete samples $\{f(t_n)\}_{n=-\infty}^{\infty}$ via $f(t) = \sum_n G(t, t_n)f(t_n)$ where $t_{n+1} - t_n = 1/(2B)$ and $G(t, t_n) = \text{sinc}(2B(t - t_n))$. By observing that the Fourier transforms of such B -bandlimited functions are invariant under the derivative operator $i\frac{d}{dt}$, we interpret the sampling function space as the invariant domain of the multiplication operator T where $Tf(t) = tf(t)$. The operator T is a simple symmetric operator with deficiency indices $(1, 1)$. In the special case of Shannon, the self-adjoint extensions of T have eigenvalues with equidistant spacing $1/(2B)$. Hence the Shannon sampling theorem is restricted to uniform sampling. By considering a general symmetric operator T , we generalize the Shannon sampling to allow non-uniformly distributed points $\{t_n\}_n$. The function space is the domain of T , and an explicit expression of the generalized $G(t, t_n)$ is obtained. In addition, a new interpolation method for N non-uniform points in a finite interval will be discussed and compared with conventional polynomial and trigonometric interpolation. (Received July 25, 2007)