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Michael E Zieve* (zieve@math.rutgers.edu). *The intersection of subfields of $K(x)$* . Preliminary report.

Let f and g be rational functions over a field K . Then the intersection $K(f) \cap K(g)$ is either K or $K(h)$ for some nonconstant $h \in K(x)$ (Lüroth/Steinitz). However, it is generally quite difficult to determine which of these occurs! For instance, there are degree-2 rational functions $f, g \in \mathbb{C}(x)$ for which the intersection is $\mathbb{C}(h)$ where $\deg(h) = 2007$. Remarkably, when f and g are polynomials and K has characteristic zero, there is a complete description of all f, g for which $K(f) \cap K(g) \neq K$ (Ritt/Schinzel). I will present joint work with Bob Beals containing results and examples in two cases: rational functions over \mathbb{C} , and polynomials over an arbitrary field. I will also discuss various consequences, for instance to the reducibility of variables-separated polynomials $f(x) - g(y)$, and (in joint work with Dragos Ghioca and Tom Tucker) to the classification of complex polynomials f, g for which some orbit $\{x_0, f(x_0), f(f(x_0)), \dots\}$ of f has infinite intersection with some orbit $\{y_0, g(y_0), g(g(y_0)), \dots\}$ of g . (Received June 19, 2007)