A graph $G$ is *minimally $k$-connected* if $G$ is $k$-connected and, for each edge $e \in E(G)$, $G\setminus e$ is not $k$-connected. Halin showed that a minimally $k$-connected graph has a vertex of degree $k$. The existence of vertices of degree $k$ in minimally $k$-connected graphs has proven to be very useful in studying the structure of $k$-connected graphs.

A matroid $M$ is *minimally $k$-connected* if $M$ is $k$-connected, and for every $e \in E(M)$, $M\setminus e$ is not $k$-connected. It is conjectured that every minimally $k$-connected matroid with at least $2(k-1)$ elements has a cocircuit of size $k$. For $k = 2$ and 3, Murty (1974) and Wong (1978) resolved this conjecture affirmatively. We prove that a minimally 4-connected matroid has a cocircuit of size 4 unless it is isomorphic to a special matroid with 9 elements. We also construct a counterexample to the conjecture for each $k \geq 5$.

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