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**Hung-ping Tsao\*** (hptsao@hotmail.com), 1151 Highland Drive, Novato, CA 94949. *Extensions of Bernoulli Theorem.*

Let  $S(n,k)$  be the sum of the  $k$ -th powers of the first  $n$  natural numbers. Bernoulli theorem states that  $S(n,k)$  can be expressed as a polynomial  $BP(n)$  of degree  $k+1$  in terms of Bernoulli numbers and binomial coefficients. The extension from the natural numbers to any arithmetic progression is accomplished by introducing general Bernoulli numbers defined in terms of general Stirling numbers. Another extension is to consider  $T(S,k)=BP(S)$  with the  $k$ -th "power" of  $S$  being  $S(n,k)$ . This nested sum has been studied to have some significant results. For example, the iterative method by integration for  $S(n,k)$  has been successfully extended to  $T(S,k)$ . (Received December 10, 2007)