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The first Picard-graded Betti numbers of some Cox rings.

The Cox ring of an algebraic variety X fits in the following analogy: $\text{Cox}(X)$ is to X as the ring of polynomials $k[x_0, \dots, x_n]$ is to \mathbb{P}^n .

It is known that the Cox ring of X is a polynomial ring if and only if X is Toric and that there is a large class of varieties, the so called Mori Dream Spaces, whose Cox rings are finitely generated algebras, that is, $\text{Cox}(X) = S/I$ for a homogeneous ideal I in a $\text{Pic}(X)$ -graded polynomial ring S .

The question of describing the ideal I and of understanding how it relates with the geometry of the variety is a fundamental open problem. The purpose of this talk is to introduce a tool to investigate this question. We define complexes of vector spaces whose homology determines the $\text{Pic}(X)$ -graded Betti numbers of $\text{Cox}(X)$ and we show that these complexes can be studied with purely geometric methods.

As an application of this technique we have proven a conjectural description, due to Batyrev and Popov, of the Cox rings of all Del Pezzo surfaces.

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