

1038-60-39

**Richard C. Bradley\*** (bradleyr@indiana.edu), Department of Mathematics, Indiana University, Bloomington, IN 47405. *A strictly stationary, “causal,” 5-tuplewise independent counterexample to the central limit theorem.* Preliminary report.

This is a strengthened version of a result that was announced earlier by the author [Abstracts of AMS 27 (2006) 608, Abstract 1020-60-35]. There exists a strictly stationary sequence  $X := (X_k, k \in \mathbf{Z})$  of random variables with the following properties: (i) The random variables  $X_k$  take just the values  $-1$  and  $1$ , with  $P(X_0 = -1) = P(X_0 = 1) = 1/2$ . (ii) For every five distinct integers  $k(1), k(2), k(3), k(4)$ , and  $k(5)$ , the five random variables  $X_{k(1)}, X_{k(2)}, X_{k(3)}, X_{k(4)}$ , and  $X_{k(5)}$  are independent. (iii) The sequence  $X$  is “causal” (and hence Bernoulli); that is,  $X_k = f(Z_k, Z_{k-1}, Z_{k-2}, \dots)$  *a.s.*, where the random variables  $Z_k, k \in \mathbf{Z}$  are i.i.d. and real-valued and the function  $f$  is Borel. (iv) The double tail  $\sigma$ -field of  $X$  is trivial (its events have probability 0 or 1). (v) For every infinite set  $Q \subset \mathbf{N}$ , there exist an infinite set  $T \subset Q$  and a nondegenerate, non-normal probability measure  $\mu$  on  $\mathbf{R}$  such that  $S_n/\sqrt{n}$  converges in distribution to  $\mu$  as  $n \rightarrow \infty, n \in T$ . (Here  $S_n := X_1 + X_2 + \dots + X_n$ .) (Received January 13, 2008)