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*Rationality, irrationality, and Wilf equivalence in the generalized factor order.* Preliminary report.

Let  $P$  be a poset and  $P^*$  be the set of all words over  $P$ . Define the generalized factor order on  $P^*$  by letting  $u \leq w$  if there is a factor  $w'$  of  $w$  such that  $|u| = |w'|$  and  $u \leq w'$  where the comparison of  $u$  and  $w'$  is done componentwise using the partial order in  $P$ . One obtains the ordinary factor order by insisting that  $u = w'$ . Given  $u \in P^*$ , we prove that the language  $\mathcal{F}(u) = \{w : w \geq u\}$  is accepted by a finite state automaton. If  $P$  is finite, it follows that the generating function  $F(u) = \sum_{w \geq u} w$  is rational. This is an analogue of a result of Björner and Sagan for the generalized subword order. Björner found a recursive formula for the Möbius function of the ordinary factor order. We show that there are finite  $P$  and  $u \in P^*$  such that the generating function  $M(u) = \sum_{w \geq u} |\mu(u, w)|w$  is not rational.

We also consider the positive integers  $\mathbb{P}$  under the usual order. In this case, one obtains a generating function  $F(u; t, x)$  by substituting  $tx^n$  each time  $n$  appears in  $F(u)$ . We show that  $F(u; t, x)$  is always rational. For  $u, v \in \mathbb{P}^*$ , we say that  $u$  is Wilf equivalent to  $v$  iff  $F(u; t, x) = F(v; t, x)$ . We give combinatorial proofs of various Wilf equivalences. (Received March 08, 2008)