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**Richard Montgomery** and **Alex L Castro\*** (alcastro@ucsc.edu), Mathematics Department - UCSC, 194 Baskin Engineering, Santa Cruz, CA 95064. *Cartan-Fefferman chains of  $SU(2)$ -invariant CR structures on  $S^3$ .*

We compute the chains associated to the left-invariant CR structures on the three-sphere. These structures are characterized by a single real modulus  $a$ . For the standard structure  $a = 1$ , the chains are well-known and are closed curves. We show that for almost all other values of the modulus  $a$  either two or three types of chains are simultaneously present : (I) closed curves, (II) quasi-periodic curves dense on two-torii, or (III) chains homoclinic between closed curves. For  $1 < a < \sqrt{3}$  no curves of the last type occur. A bifurcation occurs at  $a = \sqrt{3}$  and from that point on all three types of chains are guaranteed to exist, and exhaust all chains. The method of proof is to use the Fefferman metric characterization of chains, combined with tools from geometric mechanics. The key to the computation is a reduced Hamiltonian system, similar to Euler's rigid body system, and depending on  $a$ , which is integrable. (Received June 26, 2008)