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**Anne Fey-den Boer** and **Lionel Levine\*** (levine@math.mit.edu), Department of Mathematics, 77 Massachusetts Ave., Cambridge, MA 02139. *Fixed-Energy Sandpiles and the Poincaré Rotation Number*. Preliminary report.

In the fixed-energy sandpile on the complete graph  $K_n$ , a vertex is unstable if it has at least  $n$  chips, and topples by sending one chip to every vertex of  $K_n$ . During each time step, all unstable vertices topple simultaneously; there is no “sink,” so the total number of chips is conserved.

We study how the activity (average number of topplings per time step) depends on the number of chips in the system. Surprisingly, the activity does not increase smoothly with the number of chips, but remains constant over long intervals, punctuated by sudden jumps. In the large  $n$  limit we find a “devil’s staircase” dependence of activity on the chip density. We prove this by reducing the sandpile dynamics to iteration of a self-map of the circle  $S^1$ . The activity of the sandpile state corresponds to the Poincaré rotation number of the circle map. (Received August 06, 2008)