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Omer Angel, Abie Flaxman, David B. Wilson* (dbwilson@microsoft.com) and **Riccardo Zecchina**. *Minimum bounded depth spanning trees and Steiner trees.*

In the complete graph on n vertices, when each edge has a uniformly random weight between 0 and 1, Frieze proved that the minimum spanning tree has weight tending to $\zeta(3) = 1 + 1/2^3 + 1/3^3 + \dots$ as $n \rightarrow \infty$. We consider spanning trees constrained to have depth bounded by k . We prove that if $\Delta = k - \log_2 \log n$ tends to ∞ , the minimum bounded depth spanning tree still has weight tending to $\zeta(3)$, and that otherwise the weight is doubly-exponentially large in $-\Delta$. The minimum bounded depth spanning tree problem is NP-hard, but when $\Delta \rightarrow -\infty$, a simple greedy algorithm is asymptotically optimal, and when $\Delta \rightarrow \infty$, an algorithm which makes small changes to the minimum (unbounded depth) spanning tree is asymptotically optimal. We prove similar results for minimum bounded depth Steiner trees, where the tree must connect a specified set of m vertices, and may or may not include other vertices. In particular, when $m = \text{const} \times n$, if $k > \log_2 \log n + \omega(1)$, the minimum bounded depth Steiner tree has essentially the same weight as the minimum Steiner tree, and if $k < \log_2 \log n - \omega(1)$, the weight tends to $(1 - 2^{-k})\sqrt{8m/n}[\sqrt{mn}/2^{k-1/2}]^{1/(2^k-1)}$ in both expectation and probability. (Received August 13, 2008)