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Let $\text{mad}(G)$ denote the maximum average degree (over all subgraphs) of G and let $\chi_i(G)$ denote the injective chromatic number of G (in an injective coloring, vertices must receive distinct colors if they have a common neighbor). If Δ denotes the maximum degree of G , then clearly $\chi_i(G) \geq \Delta$. We study upper bounds on $\text{mad}(G)$ that imply $\chi_i(G) \leq \Delta + c$ for $c \in \{0, 1, 2\}$. In particular, we have the following results.

If $\text{mad}(G) < \frac{14}{5}$ and $\Delta \geq 4$, then $\chi_i(G) \leq \Delta + 2$. When $\Delta = 3$, we show that $\text{mad}(G) < \frac{36}{13}$ implies $\chi_i(G) \leq 5$; in contrast, we give a graph G with $\Delta = 3$, $\text{mad}(G) = \frac{36}{13}$, and $\chi_i(G) = 6$.

If $\text{mad}(G) \leq \frac{5}{2}$, then $\chi_i(G) \leq \Delta + 1$; similarly, if $\text{mad}(G) < \frac{42}{19}$, then $\chi_i(G) \leq \Delta$. When G is a planar graph with $\Delta \geq 4$, we have the following improvements. If $\text{girth}(G) \geq 9$, then $\chi_i(G) \leq \Delta + 1$; similarly, if $\text{girth}(G) \geq 13$, then $\chi_i(G) = \Delta$. (Received January 26, 2009)