

1047-05-267

**Philippe R Di Francesco\*** ([philippe.di-francesco@cea.fr](mailto:philippe.di-francesco@cea.fr)), CEA Saclay/ IPhT, 91191 Gif sur Yvette, France. *Q-systems and Cluster Positivity.*

Q-systems are integrable systems of recursion relations first introduced in the study of quantum spin chains based on Lie groups. These were interpreted recently [Kedem07]-[DiFrancesco-Kedem07] as mutation relations for certain cluster algebras. The latter were introduced by Fomin and Zelevinsky as discrete dynamical systems describing the evolution by mutation of (cluster) variables, with the built-in property that any mutated variable may be expressed as a Laurent polynomial of any other cluster variable. It was conjectured that the corresponding Laurent polynomials have non-negative integer coefficients.

In the present work [DiFrancesco-Kedem08], we prove this positivity property for the cluster algebras associated to the  $A_r$  Q-systems. This is done with the help of integrability by giving an explicit combinatorial expression for the cluster variables as partition functions for positively weighted paths on finite target graphs. Cluster mutation can then be understood as continued fraction rearrangements for the corresponding generating functions. We also present an alternative formulation in terms of domino tilings of plane domains, including possible defects. (Received January 30, 2009)