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*Noncrossing partitions and the shard intersection order.*

I will discuss the shard intersection order  $(W, \preceq)$  on a finite Coxeter group  $W$ . This poset is a lattice and has the noncrossing partition lattice  $\text{NC}(W)$  as a sublattice. This new construction of  $\text{NC}(W)$  yields a new proof that  $\text{NC}(W)$  is a lattice. The shard intersection order is graded and atomic. Its rank generating function is the  $W$ -Eulerian polynomial. Many order-theoretic properties of  $(W, \preceq)$ , like Möbius number, number of maximal chains, etc., are analogous to corresponding properties of  $\text{NC}(W)$ .

The shard intersection order is most naturally defined in terms of the polyhedral geometry of the reflecting hyperplanes of  $W$ , and in particular certain codimension-1 polyhedral cones called shards. The reflecting hyperplanes are cut into shards according to a simple rule. Shards were originally defined as a way of understanding lattice congruences of the weak order on  $W$ . The collection of arbitrary intersections of shards forms a lattice under reverse containment. Arbitrary intersections of shards are in bijection with elements of  $W$ , so the lattice of shard intersections defines a partial order “ $\preceq$ ” on  $W$ . I will illustrate the definitions and results with a running example, taking  $W$  to be the symmetric group  $S_4$ . (Received February 02, 2009)