

1047-05-414

**Luis A Goddyn\*** (goddyn@math.sfu.ca), Mathematics, Simon Fraser University, Burnaby, BC V5A 1S6, Canada. *The rank-chromatic number and the rank-flow number.* Preliminary report.

A new family of graph chromatic numbers and flow numbers arises as a restriction to graphs of a natural matrix optimization problem. For any graph  $G$  and positive integer  $k$  we define the  $k$ -rank-flow number of  $G$  to be

$$\phi_k(G) = \min_{\vec{G}} \max_P \frac{d(P)}{d^+(P)} \in \mathbb{Q} \cup \{\infty\}.$$

Here  $\vec{G}$  ranges over the orientations of  $G$ , and  $P$  ranges over the ordered partitions  $(V_0, V_1, \dots, V_k)$  of  $V(\vec{G})$  into  $k+1$  parts. Also  $d(P)$  is the number of arcs of  $\vec{G}$  whose ends lie in distinct parts of  $P$ , and  $d^+(V)$  is the number of those arcs, say  $uv$  with  $u \in V_i, v \in V_j$ , for which  $i < j$ .

Dually, we may define the  $k$ -rank-chromatic number, in terms of strong orientations  $P$  of all the subgraphs of  $G$  having Betti number  $k$  (details omitted).

Then  $\phi_1(G)$  is just the circular flow index of  $G$ , and  $\chi_1(G)$  is just the circular chromatic number of  $G$ . The *rank-chromatic sequence*  $[\chi_1(G), \chi_2(G), \dots, \chi_{n-1}(G)]$  carries significantly more information than the ordinary chromatic number does, and similar for the *rank-flow sequence*. I will describe the connection to geometry, and determine some values and bounds for these invariants. (Received February 02, 2009)